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COMMENT

Real-space renormalisation group approach to random walks on self-avoiding walks

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Abstract. The t dependence of the average of the end-to-end distance $\langle R_t^2 \rangle$ of t-step random walks on self-avoiding walks is studied by small cell real-space renormalisation. The results are in qualitative agreement with our earlier Monte Carlo data.

In a recent letter (Chowdhury and Chakrabarti 1985a, from now on referred to as I) we investigated the phenomenon of the random walk (RW) on self-avoiding walks (SAWS). The average of the square of the end-to-end distance $\langle R_t^2 \rangle$ of t-step RWS on N-step SAWS was shown to be of the form

$$\langle R_t^2 \rangle \sim N^{\delta} t^{k} \tag{1}$$

where $\delta \simeq 0.1$ and $k \simeq 0.72$ in two dimension, provided the end-to-end distance of the RWS is much less than that of the SAW. Both δ and k were estimated from Monte Carlo (MC) simulation of the RW on SAW. The value of δ observed was further supported by our real-space renormalisation group (RSRG) result for the resistance of SAW chains (see I). If only hops along the SAW chain were allowed one would expect k in d dimension to be equal to ν_s, the end-to-end distance exponent for SAWS in d dimension. But the smaller value of k observed in our MC simulation (remember that $\nu_s = 0.75$ in $d = 2$) was argued to be a consequence of the hoppings across the so-called 'bridges' (Helman *et al* 1984, Yang *et al* 1985 and I). However, no RSRG result was presented in support of this claim. For convenience let us denote the value of k, when hops along the 'bridges' are allowed, as k_1 and that when hops along the 'bridges' are forbidden as k_2. The aim of this comment is to show that although effective medium theory (EMT) yields $k_1 = k_2$ small cell RSRG indicates $k_1 < k_2$, in qualitative agreement with our MC result in I.

Let $P_n(t)$ be the probability that the random walker is at the site n at time t. Then proceeding as in I, we get, in the EMT

$$\mathrm{d}\langle P_n \rangle / \mathrm{d}t = T_{\mathrm{eff}}(\langle P_{n+1} \rangle + \langle P_{n-1} \rangle - 2\langle P_n \rangle) \tag{2}$$

where the quantity $\langle X \rangle$ corresponds to the EMT value of X, and T_{eff} is the effective hopping probability. In other words, equation (2) describes RW on an effectively one-dimensional lattice with effective hopping probability T_{eff}. Now, defining the

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generating function (Van Kampen 1981)

$$F(z, t) = \sum_n z^n \langle P_n \rangle$$

and using the relations

$$[\partial \ln F(z, t)/\partial z]_{z=1} = \langle R_t \rangle$$

and

$$[\partial^2 \ln F(z, t)/\partial z^2]_{z=1} = \langle R_t^2 \rangle - \langle R_t \rangle^2 - \langle R_t \rangle$$

together with equation (1) we get $\langle R_t^2 \rangle \sim t$ for distance R_t measured along the SAW chain. Therefore, the corresponding distance measured in the square lattice (the embedding Euclidean lattice) is $\langle R_t^2 \rangle \sim t^{0.75}$, i.e. $k_1 = k_2$. Thus, within EMT, diffusion along the bridges has no effect on the values of the relevant exponents.

Our RSRG treatment of the RW on SAW has been carried out following the earlier RSRG approaches to RW in various different contexts (Gould *et al* 1983, Family and Gould 1984, Sahimi and Jerauld 1983a, b, Sahimi *et al* 1984). We shall consider only square lattice cells of linear dimension $b = 2$ (see figure 2(b) of I). Those links across which hoppings of the random walker are allowed will be called resistors, other links will be called insulators. The renormalisation scheme consists of the following rules.

(i) All the SAW configurations start from the south-west corner of the cell and end at either of the two northernmost end points; a weight factor f^N is associated with each of the N-step SAW, where f is the corresponding fugacity (see, e.g., Stanley *et al* 1982).

(ii) All the SAW configurations allowed by rule (i) are rescaled to a SAW of single step along the north-south axis on the rescaled lattice (rules (i) and (ii) were also followed in the RSRG treatment of the resistance of SAWs in I).

(iii) All the RWs also start from the south-west corner of the cell and end at either of the two northernmost end points; the differences between the rules (i) and (iii) being that (a) RW hops only along the resistors are allowed, hops along the insulators being forbidden, and (b) self-intersections of the RW are allowed and, hence, the number of RWs can, in principle, be infinitely large.

(iv) Since at the critical fugacity only RWs of step size $t \leq (\text{end-to-end distance})^2$ are important (Gould *et al* 1983, Family and Gould 1984, Sahimi and Jerauld 1983a, b, Sahimi *et al* 1984) we shall not consider RWs of more than five steps. A weight factor w^t is associated with each of the t-step ($t \leq 5$) RW, where w is the corresponding fugacity.

(v) All the RW configurations allowed by rules (iii) and (iv) are rescaled to a RW of single step along the north-south axis on the rescaled lattice.

We shall consider two cases:

case I: both the streets and the bridges are resistors, and

case II: the bridges are assumed to be insulators, the only resistors being the streets.

Following the rules (i)–(v), the renormalised fugacity f' is given by

$$f' = f^2 + 2f^3 + f^4 \tag{3}$$

both in case I and case II. The corresponding renormalised fugacity w' is given by

$$w'f' = f^2(w^2 + 2w^4) + 2f^3(w^3 + 3w^5) + f^4(w^2 + 5w^4) \tag{4}$$

in case I, and

$$w'f' = f^2(w^2 + 2w^4) + 2f^3(w^3 + 3w^5) + f^4 w^4 \tag{5}$$

in case II. The fixed point of (3) is given by $f^* = 0.466$ and leads to $\nu_s = \ln 2/\ln(df'/df)_{f^*} = 0.715$; the latter value being quite close to the exact value $\nu_s = 0.75$ in $d = 2$. The fixed point corresponding to (4) is $w^* = 0.605$ and yields $k_1 \simeq 0.83$. This is about 15% higher than the corresponding MC result, $k_1 \simeq 0.72$ (see I). The fixed point corresponding to (5) is $w^* = 0.65$ and leads to $k_2 \simeq 0.86$. The latter value is also about 15% higher than the exact value $k_2 = 0.75$ and about 20% higher than the expected value 0.715. The errors of k_1 and k_2 arise from the inadequacy of the small-cell renormalisation. For example, in the case of the RW on square lattice (see Sahimi *et al* and references therein) such small-cell renormalisation yields the end-to-end distance exponent $\nu_{RW} = 0.585$ which is about 17% higher than the exact value $\nu_{RW} = 0.5$. However, the aim of the present work is not to compute accurate values of k but to find the universality classes. Thus, we find that the two exponents k_1 and k_2 are different. We emphasise that the small-cell RSRG result reported here provides only a plausible argument in support of our earlier MC result that $k_1 < k_2$. The minimum size of a square lattice cell required for the detailed study of the flow diagram is $b = 3$. This, in turn, requires enumeration of all possible RWs with up to $t = 13$ steps. However, MC renormalisation enables one to renormalise much bigger cells. Results of work along these lines (Chowdhury and Chakrabarti 1985b) will be reported elsewhere.

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